

## **Fuzzy Control of High-Accuracy Thermostatic Device in Double-Walled Container**

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This paper is concerned with temperature control of double-walled thermostatic container, which is one of the most important element in the measuring device of barometric differences. In order to measure the barometric differences with high accuracy, it is necessary for the container to be kept at constant temperature within the accuracy of  $0.01^{\circ}\text{C}$ . But, due to random disturbances based on the fan and ambient air temperature, the temperature in the container suffers an unfavorable influence on control accuracy. In order to realize a simple control strategy and high accuracy, the fuzzy controller is applied and, through simulation studies, the validity is clarified.

### **1. Introduction**

This study develops the control strategy to keep at a constant temperature with high accuracy for a double-walled thermostatic container, which is one of the most important elements in the measuring device of the barometric differences.<sup>1)</sup> Now, if we demand the measuring accuracy within  $\pm 0.3$  Pa in the device, it is necessary for the container to be kept at a constant temperature with the accuracy of  $0.01^{\circ}\text{C}$ . For this purpose, we had already proposed a thermostatic device with the double-walled container.<sup>2)</sup>

One of problems in this device is that the thermostatic container is always subjected to the effect of the ambient temperature. Also, due to the fan to uniform the temperature in the space, the random noise comes into existence and suffers an unfavorable influence to the temperature characteristics. The another problem is that it takes 2 or 3 hours to arrive at a desired constant temperature by the on-off regulator.

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To improve these response characteristics, the fuzzy controller is applied and the effectiveness clarified through simulation studies.

## 2. Structure of Thermostatic Device

The structure of thermostatic device is shown in Fig. 1, which consists of a double-walled container. The outer casing is made of styrene form and the inner one the special glass covered by the styrene form. The intermediate space between two casings can be thermostated at 50°C by the regulation of the heater.

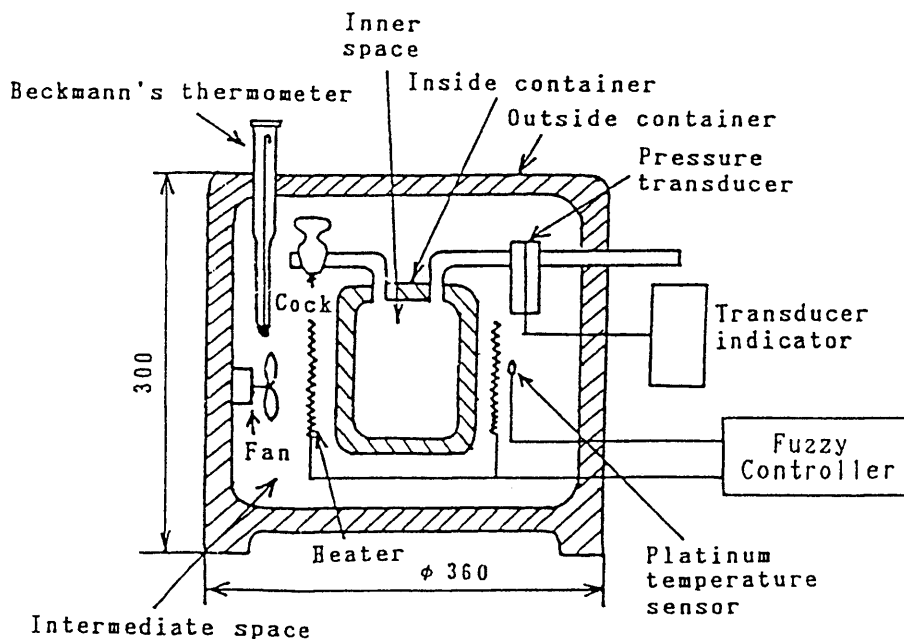


Fig. 1 Structure of a Thermostatic Device

From this regulation and heat transfer characteristics of the inner vessel, the inner space can be structurally kept at a constant temperature with high accuracy, compared with the intermediate space. In the intermediate space, the small fan to uniform the temperature in the space, the thermometer and pressure transducer to detect the difference of pressure between the ambient air and the one in the inner space are settled. The temperature in the intermediate space can be detected by a thermometer and transformed into voltage through A/D converter. This voltage is given as an input to the control circuit in the computer and the fuzzy inference can be carried out. Based on the results of fuzzy inference, the heater output can be calculated in the computer.

Through D/A converter, the heater can be regulated and the temperature in the intermediate space can be controlled. Thus, the inner space can be controlled at 50°C with high accuracy of 0.01°C. For the temperature control system of double-walled thermostated container, the mathematical model of discrete-time may be given as<sup>3)</sup>

$$\theta(k) = 1.991\theta(k-1) - 0.991\theta(k-2) + 0.08dq(k-1-\mu) - 0.08dq(k-2-\mu) + \xi(k), \quad (1)$$

where  $\theta$  is the temperature of the intermediate space,  $dq$  the output of the heater,  $\mu$  a delay time of a temperature sensor and  $\xi$  a random noise caused by the air stirring of fan in the intermediate space.

### 3. Fuzzy Control and Fuzzy Inference

**3.1 Fuzzy Inference** The block diagram of thermostatic system with a fuzzy controller is shown in Fig. 2.

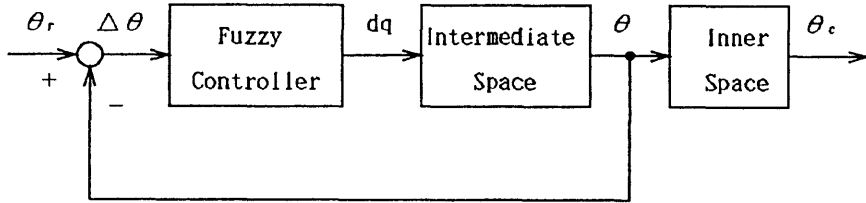


Fig. 2 Block Diagram of Thermostatic Device with Fuzzy Controller

Suppose that pre-conditioned variables are the output error  $\Delta\theta$  of the thermostatic device and its derivative  $\dot{\Delta\theta}$  at one sampling time. Also, the post-conditioned variable is the manipulated volume  $dq$  which implies the fever quantity of the heater at unit time. Letting  $\theta_n$  be the output temperature at time  $n$ ,  $\Delta\theta_n$  the error for the desired temperature,  $q_n$  the manipulating quantity and  $\theta_r$  the desired value, the following relations are defined;  $\Delta\theta_n = \theta_r - \theta_n$ ,  $\dot{\Delta\theta} = \theta_{n-1} - \theta_n$  and  $dq = q_n - q_{n-1}$ .

Also, the  $\theta_c$  in Fig. 2 is the temperature in the inner space. Then, the control strategy of fuzzy inference can be described as follows.<sup>4)</sup>

$$R_i: \text{ if } \Delta\theta \text{ is } H_{ai} \text{ and } \dot{\Delta\theta} \text{ is } H_{bi}, \text{ then } dq \text{ is } H_{ci} \quad (i=1,2,\dots,m) \quad (2)$$

Here, the  $i$  is the number of control strategy,  $H_{ai}$  and  $H_{bi}$  the pre-conditioned fuzzy variables,  $H_{ci}$  the post-conditioned fuzzy variable and  $m$  the control number.

In order to reduce the fuzzy inference time, the simplified reasoning method may be adopted. Then, the pre- and post-conditioned fuzzy variables can be represented as (a), (b) and (c) in Fig. 3.

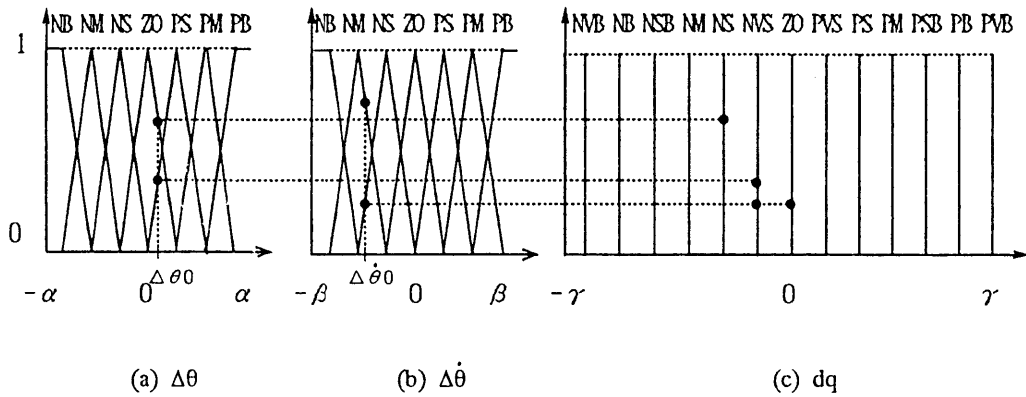


Fig. 3 Membership Functions

Here,  $\alpha$ ,  $\beta$  and  $\gamma$  are bands of pre- and post-conditioned fuzzy variables, respectively. Then, the fuzzy control rules of Eq. (2) can be mentioned as Table 1.

Table 1 Rules of Fuzzy Controller

$\Delta\theta \backslash \Delta\dot{\theta}$	N B	N M	N S	Z O	P S	P M	P B
N B	Rule 01 NVB	Rule 02 N B	Rule 03 NSB	Rule 04 N M	Rule 05 N S	Rule 06 NVS	Rule 07 Z O
N M	Rule 08 N B	Rule 09 NSB	Rule 10 N M	Rule 11 N S	Rule 12 NVS	Rule 13 Z O	Rule 14 PVS
N S	Rule 15 NSB	Rule 16 N M	Rule 17 N S	Rule 18 NVS	Rule 19 Z O	Rule 20 PVS	Rule 21 P S
Z O	Rule 22 N M	Rule 23 N S	Rule 24 NVS	Rule 25 Z O	Rule 26 PVS	Rule 27 P S	Rule 28 P M
P S	Rule 29 N S	Rule 30 NVS	Rule 31 Z O	Rule 32 PVS	Rule 33 P S	Rule 34 P M	Rule 35 PSB
P M	Rule 36 NVS	Rule 37 Z O	Rule 38 PVS	Rule 39 P S	Rule 40 P M	Rule 41 PSB	Rule 42 P B
P B	Rule 43 Z O	Rule 44 PVS	Rule 45 P S	Rule 46 P M	Rule 47 PSB	Rule 48 P B	Rule 49 PVB

In Table 1, the notation of fuzzy sets implies,

N ... Negative , P ... Positive , ZO ... Zero , B ... Big ,

M ... Medium , S ... Small , V ... Very .

For example, the NVS is negative and very small. That is, the Rule 24 implies the following,

If  $\Delta\theta$  is ZO and  $\Delta\dot{\theta}$  is NS , then dq is NVS.

When both the error and the rate error are given by  $\Delta\theta_0$  and  $\Delta\dot{\theta}_0$ , the estimated value  $dq_0$  may be obtained as the following weight average by the simplified method,

$$dq_0 = \sum \{H_{ci}(\Delta\theta_0, \Delta\dot{\theta}_0) \cdot \omega_i\} / \sum \omega_i , \quad (3)$$

where  $\omega_i$  is given by

$$\omega_i = \text{Min}\{H_{ni}(\Delta\theta_0), H_{bi}(\Delta\dot{\theta}_0)\} . \quad (4)$$

The membership functions of the input  $\Delta\theta$  are, when the input  $x$  is added, given by the following equations;

$$NB: H_a(x) = \begin{cases} 1 & ; & x \leq -\alpha \\ -3x/\alpha - 2 & ; & -\alpha < x \leq -2\alpha/3 \end{cases} \quad (5)$$

$$NM: H_a(x) = \begin{cases} 3x/\alpha + 3 & ; & -\alpha < x \leq -2\alpha/3 \\ -3x/\alpha - 1 & ; & -2\alpha/3 < x \leq -\alpha/3 \end{cases} \quad (6)$$

$$NS: H_a(x) = \begin{cases} 3x/\alpha + 2 & ; & -2\alpha/3 < x \leq -\alpha/3 \\ -3x/\alpha & ; & -\alpha/3 < x \leq 0 \end{cases} \quad (7)$$

$$ZO: H_a(x) = \begin{cases} 3x/\alpha + 1 & ; & -\alpha/3 < x \leq 0 \\ -3x/\alpha + 1 & ; & 0 < x \leq \alpha/3 \end{cases} \quad (8)$$

$$PS: H_a(x) = \begin{cases} 3x/\alpha & ; & 0 < x \leq \alpha/3 \\ -3x/\alpha + 2 & ; & \alpha/3 < x \leq 2\alpha/3 \end{cases} \quad (9)$$

$$PM: H_a(x) = \begin{cases} 3x/\alpha - 1 & ; & \alpha/3 < x \leq 2\alpha/3 \\ -3x/\alpha + 3 & ; & 2\alpha/3 < x \leq \alpha \end{cases} \quad (10)$$

$$PB: H_a(x) = \begin{cases} 3x/\alpha - 2 & ; & 2\alpha/3 < x \leq \alpha \\ 1 & ; & \alpha < x \end{cases} \quad (11)$$

As the membership functions of another input  $\Delta\dot{\theta}$  is the same as the case of the input  $\Delta\theta$ , the membership functions  $H_b(x)$  are obtained by replacing  $\alpha$  with  $\beta$ .

As an illustrative example, the calculation approach is shown in the domain that  $0 \leq \Delta\theta_0 \leq \alpha/3$  and  $-2\beta/3 \leq \Delta\dot{\theta}_0 \leq -\beta/3$ , as shown in Fig. 3. In Eqs. (8) and (9),  $\Delta\theta_0$  is both ZO and PS, and also, in Eqs. (6) and (7),  $\Delta\dot{\theta}_0$  is both NM and NS, which correspond to rules 23, 24, 30 and 31. In each rules, both the membership functions  $H_{ai}(\Delta\theta_0)$  and  $H_{bi}(\Delta\dot{\theta}_0)$  are given as follows;

$$\left. \begin{aligned} H_{a23}(\Delta\theta_0) &= H_{a24}(\Delta\theta_0) = -3 \cdot \Delta\theta_0 / \alpha + 1 \\ H_{b23}(\Delta\dot{\theta}_0) &= H_{b24}(\Delta\dot{\theta}_0) = -3 \cdot \Delta\dot{\theta}_0 / \beta - 1 \end{aligned} \right\} \quad (12)$$

$$\left. \begin{aligned} H_{a30}(\Delta\theta_0) &= H_{a31}(\Delta\theta_0) = 3 \cdot \Delta\theta_0 / \alpha \\ H_{b30}(\Delta\dot{\theta}_0) &= H_{b31}(\Delta\dot{\theta}_0) = 3 \cdot \Delta\dot{\theta}_0 / \beta + 1 \end{aligned} \right\} \quad (13)$$

In Eq. (4), the  $\omega_i$  is determined as the minimum value between  $H_{ai}$  and  $H_{bi}$ . As an example, the values of both  $\omega_{30}$  and  $H_{c30}$  are calculated as shown in Fig. 4. Substituting  $\omega_i$  into Eq. (3), the  $dq_0$  is obtained.

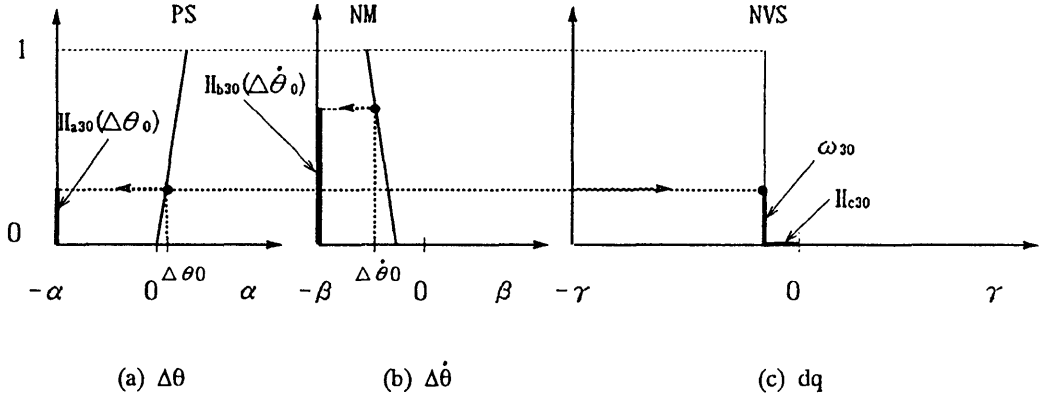


Fig. 4 Example of  $\omega_{30}$  and  $H_{c30}$

**3.2 Simulation Experiments** Using the model of Eq. (1), the simulation experiments of the fuzzy control system have been performed. The simulation result is shown in Fig. 5, in which (a) shows the temperature in the intermediate space and (b) the temperature in the inner space. In this case, the desired temperature is about 50°C and the ambient air temperature 13°C. Also,

the maximum output of heater is 10 w and the delay time of temperature sensor is  $\mu=2$ . The random noise due to the air stirring of a fan in the intermediate space is, from the experimental result, assumed to be a white Gaussian noise with the mean  $m=0$  and the variance  $\sigma^2=0.03$ . Concerning with the control accuracy of the temperature, the desired control error of the temperature must be held within  $0.01^\circ\text{C}$  in the inner space. For this purpose, it is necessary for the variance of inner temperature to be within 0.000638. The parameters of the fuzzy control device are set as  $\alpha=1$ ,  $\beta=5$  and  $\gamma=10$ . The results of simulation experiments are shown as the responses (1) of (a) and (b) in Fig. 5. In this case, though the response satisfies the desired control accuracy in the steady state, the transient response is not sufficient. This implies that the optimal values of  $\alpha$ ,  $\beta$  and  $\gamma$  must be explored. Here, we describe the method to determine these values by the optimization method.<sup>9)</sup>

**3.3 Optimization of Fuzzy Control System** In order to optimize the fuzzy control system, the least square method can be, for simplicity, used as the evaluation function.

For real function of simultaneous equations,

$$f_i(x_1, x_2, \dots, x_n) = 0 \quad , \quad i=1, 2, \dots, p \ ( \geq n) \quad (14)$$

the evaluation function  $S$  is given as

$$S(x_1, x_2, \dots, x_n) = \sum_{i=1}^p f_i^2(x_1, x_2, \dots, x_n) \quad (15)$$

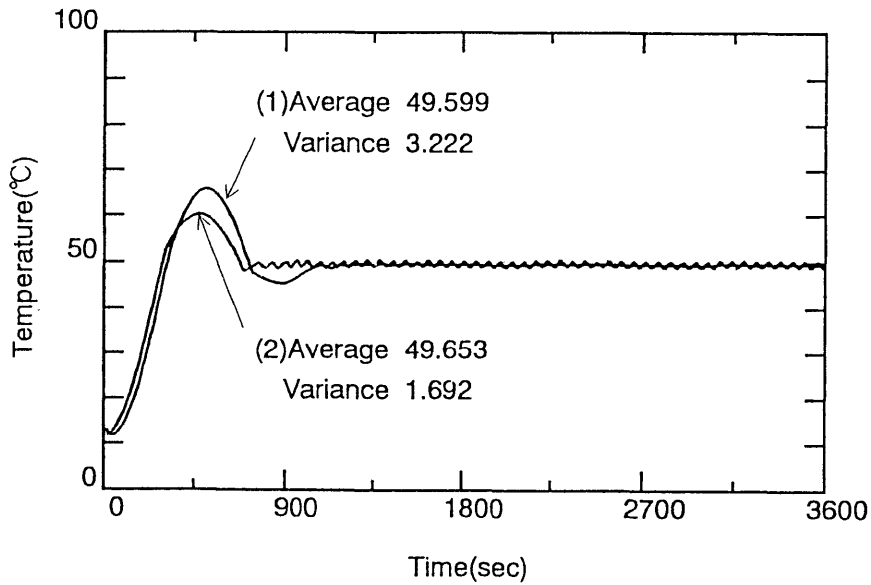
Here, we set  $F=[f_1, f_2, \dots, f_p]$  and  $X=[x_1, x_2, \dots, x_n]$ . The gradient of  $F$  by the  $k$ -th repeated calculation may be given as follows,

$$F_{X(k)} = \left( \frac{\partial f_i(X_{(k)})}{\partial x_j} \right) \quad (i=1, 2, \dots, p ; j=1, 2, \dots, n) \quad (16)$$

In the repeated calculation, if the inverse matrices of  $F_{X(k)}^T$  and  $F_{X(k)}$  exist, the following series  $X_{(1)}, X_{(2)}, \dots, X_{(k+1)}$  converges, for arbitrary initial value  $X_{(0)}$ , to the stationary value, which optimize the function  $S$ .

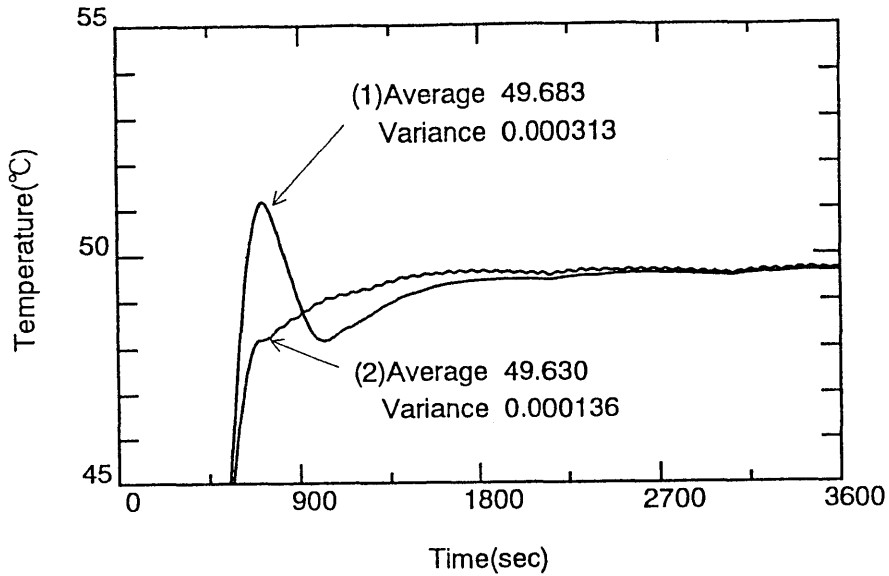
$$X_{(k+1)} = X_{(k)} - \zeta \cdot \Delta X_{(k)} \\ X_{(k)} - \zeta \cdot ((F_{X(k)}^T F_{X(k)})^{-1} F_{X(k)}^T F(X_{(k)})) \quad (17)$$

Here, the value  $\zeta$  satisfies the relation;  $S(X_{(k)} - \zeta \Delta X_{(k)}) \leq (1 - \zeta \lambda) S(X_{(k)})$ , where  $\lambda$  is an arbitrarily selected positive number. In order to determine the values of  $\alpha$ ,  $\beta$  and  $\gamma$ , the above optimization approach can be applied. For the optimization problem without constrained conditions, the variables  $x_1$ ,  $x_2$ , and  $x_3$  may be represented by forms of  $\alpha=10^{x_1}$ ,  $\beta=10^{x_2}$  and  $\gamma=10^{x_3}$ . Using these index numbers, the gradient of  $F$  can be calculated by Eq. (16). Now, letting initial values be  $x_{1(0)}=0$ ,  $x_{2(0)}=0.7$  and  $x_{3(0)}=1$ , the values of the  $k+1$ -th variables are obtained. By the repeated calculations, the results of optimization can be explored as  $x_1=-0.3545$ ,  $x_2=0.8905$  and  $x_3=1.2159$ . Accordingly, the optimal values of membership functions are determined  $\alpha^*=0.6918$ ,  $\beta^*=7.7721$  and  $\gamma^*=16.4395$ . Using these parameters, the simulation experiments have been performed, which results have been shown as the responses (2) in both (a) and (b) of Fig. 5. Comparing with these two results of (1) and (2) in Fig. 5, it is ascertained that the variance of (2) is by the application of optimization method, satisfactory from the viewpoints of the control accuracy.



(a) Temperature in Intermediate Space





(b) Temperature in Inner Space

Fig. 5 Simulation Experiment (A)

(by Optional Parameters and Optimized Ones)

#### 4. Effects of Ambient Temperature and Random Noise

First, we shall consider the effect due to the variation of ambient temperature. Now, suppose that the temperature in the inner space was kept stationary at  $t=1200$  sec under the ambient air temperature of  $13^{\circ}\text{C}$ . Then, we examined the behaviors that the ambient air temperature changed to  $25^{\circ}\text{C}$  or  $35^{\circ}\text{C}$ . Fig. 6 shows the result of simulation experiments in the inner space. From Fig. 6, it became clear that this device was little subjected to the effect of ambient air temperature. Next, we shall consider the effect of random noise, which generates mainly by the fan in the intermediate space. Fig. 7 shows the results of simulation experiment in cases that the variances of random noise are both  $\sigma^2=0.03$  and  $0.06$ , when the ambient temperature is  $13^{\circ}\text{C}$ . From Fig. 7, it was ascertained that the temperature arrives at the stationary state so fast as the variance is small. In any cases, the variance of the temperature in the inner space was kept within  $6.38 \times 10^{-4}$ .

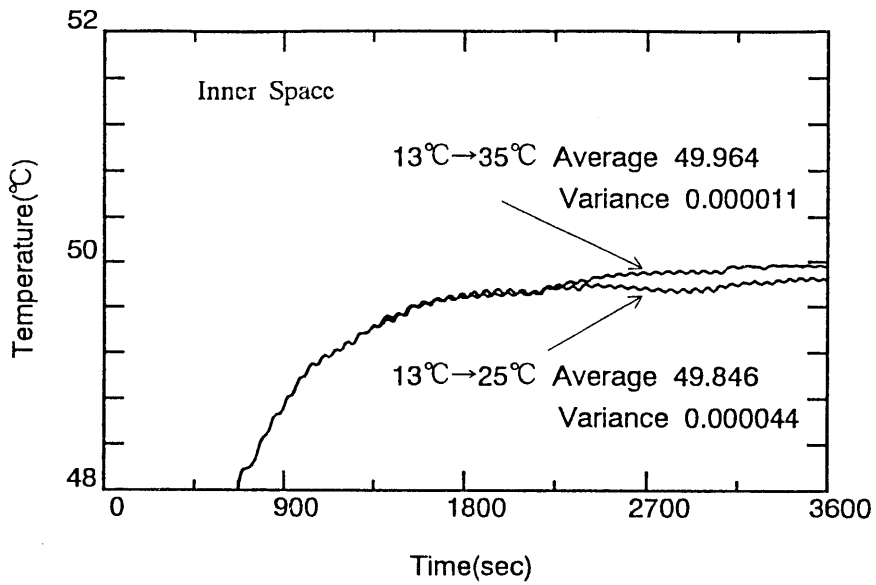


Fig. 6 Simulation Experiment (B)  
(Effect of ambient temperature)

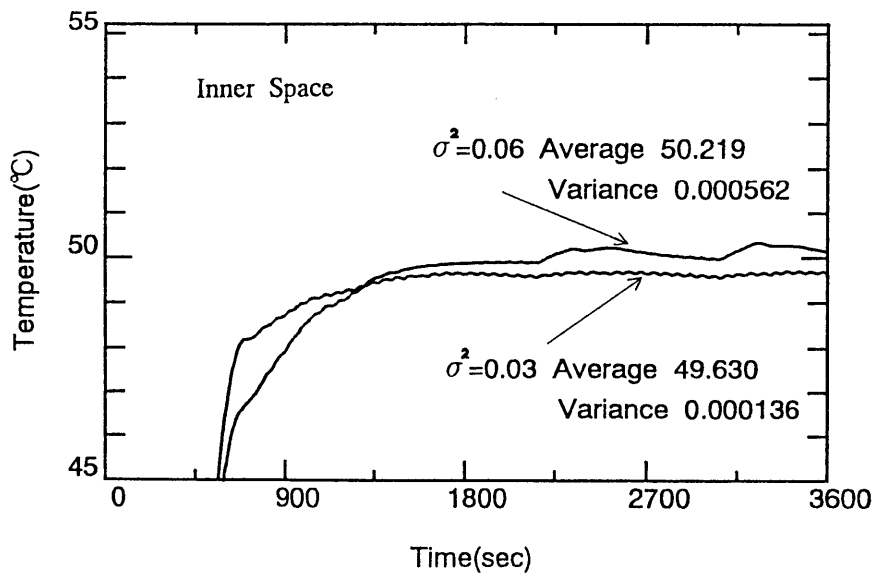


Fig. 7 Simulation Experiment (C)  
(Effect of random noise)

## 5. Conclusions

For a double-walled thermostatic container, the fuzzy temperature control strategy has been developed and its validity certified through simulation studies. The results of fuzzy control for thermostic device can be summarized as follows.

(1) In order to explore fuzzy variables  $\alpha$ ,  $\beta$  and  $\gamma$ , the least square method is utilized and the optimal values of  $\alpha$ ,  $\beta$  and  $\gamma$  are obtained. Through simulation experiments, the usefulness was ascertained.

(2) For the variations of the ambient temperature or internal random noise, the inside of the container can be kept at a constant temperature with high accuracy.

(3) Utilizing both the fuzzy control strategy and the optimization method, it is possible for the control accuracy in the inner space to be always kept within  $0.01^{\circ}\text{C}$ .

(4) It is sufficiently possible to implement this fuzzy control by on-line computations. Then, this control strategy has the possibility of practical use for the real system.

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